Dr. Virendra Swarup Education Centre, Kidwai Nagar

Class 12-C/D **Maths Practice Sheet**

PRACTICE SHEET ON DIFFERENTIATION

(a) If $f(x)=\begin{cases}\dfrac{\log(1+ax)-\log(1-bx)}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$, find 'k'.

(b) Show that $f(x)=x^2$, is differentiable at $x=1$, and hence find $f'(1)$.

(c) Discuss the differentiability of $f(x)=x|x|$ at $x=0$.

(d) Discuss the differentiability of $f(x)=\begin{cases}xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0 \text{ at } x=0\\ 0, & x=0\end{cases}$

(e) If $f(x)$ is differentiable at $x=a$, find $\lim_{x\to a}\dfrac{x^2f(a)-a^2f(x)}{x-a}$

(f) For what choice of a and b is the function $f(x)=\begin{cases}x^2, & x\le c\\ ax+b, & x>c\end{cases}$ is differentiable at $x=c$

(g) A function $f:R\to R$ satisfies the equation $f(x+y)=f(x).f(y)$ for all $x,y\in R, f(x)\neq 0$. Suppose that the function is differentiable at $x=0$ and $f'(0)=2$, prove that $f'(x)=2f(x)$

(h) Differentiate: (i) $y=\dfrac{2^x\cot x}{\sqrt{x}}$ (ii) $y=e^x\log\sqrt{x}\tan x$ (iii) $y=\dfrac{\sin x-x\cos x}{x\sin x+\cos x}$

(i) If $f(x)=|\cos x|$, find $f'\left(\dfrac{\pi}{4}\right), f'\left(\dfrac{3\pi}{4}\right)$

(j) If $f(x)=|\cos x-\sin x|$, find $f'\left(\dfrac{\pi}{6}\right), f'\left(\dfrac{\pi}{3}\right)$

(k) Differentiate wr.t. 'x' from first principles: $e^{x^2}, e^{2x}, e^{\sqrt{x}}, e^{\sin x}, e^{\sqrt{\tan x}}, e^{x\sin x}$

(l) Differentiate, w.r.t. 'x': $\sin\left(m\sin^{-1}x\right), e^{\cos^{-1}\left(\sqrt{1-x^2}\right)}, \cos(\log)^2$

(m)If $y=\sqrt{x+1}+\sqrt{x-1}$, prove that $\sqrt{x^2-1}\dfrac{dy}{dx}=\dfrac{1}{2}y$

(n) If $y=\dfrac{x}{x+2}$, prove that $x\dfrac{dy}{dx}=(1-y)y$

(o) If $y=\log\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)$, prove that $\dfrac{dy}{dx}=\dfrac{x-1}{2x(x+1)}$

(p) If $y=\sqrt{x}+\dfrac{1}{\sqrt{x}}$, prove that $2x\dfrac{dy}{dx}=\sqrt{x}-\dfrac{1}{\sqrt{x}}$

(q) If $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, prove that $\dfrac{dy}{dx} = 1 - y^2$

(r) If $y = e^x \cos x$, prove that $\dfrac{dy}{dx} = \sqrt{2} e^x \cos\left(x + \dfrac{\pi}{4}\right)$

(s) If $y = \sqrt{x^2 + a^2}$, prove that $y\dfrac{dy}{dx} - x = 0$

(t) If $xy = 4$, prove that $x\left(\dfrac{dy}{dx} + y^2\right) = 3y$

(u) If $x^m y^n = (x + y)^{m+n}$ prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$

(v) Find $\dfrac{dy}{dx}$ for the following:

(i) $x = at^2, y = 2at$

(ii) $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$

(iii) $x = a(1 - \cos\theta), y = a(\theta + \sin\theta)$ at $\theta = \dfrac{\pi}{2}$

(iv) $x = \dfrac{2t}{1 + t^2}, y = \dfrac{1 - t^2}{1 + t^2}$

(v) $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}, y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$,

(vi) $x = a\left(t + \dfrac{1}{t}\right), y = a\left(t - \dfrac{1}{t}\right)$